Replicating Robert Rosen's (M,R) Systems

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Abstract

The central interest of this paper is Robert Rosen's replicating (M,R)-Systems, presented in his book Life Itself, where M and R signify Metabolism and Repair, respectively. We look anew at Rosen's model of an organism in the light of extensive research into natural hierarchical systems, and this paper presents conclusions drawn from a comparison between Rosen's relational model of an organism and that of a birational complementary natural hierarchy. Rosen's model is 'replicated' in a number of different ways which lend credence to the argument that birationality sheds new light on the nature of life and the usefulness of his accomplishments. Careful assessment also prompts a number of questions as to the validity and comprehensiveness of the book's arguments. We accept that Rosen's relational model has provided a useful stepping stone to understanding the nature of life, but also suggest that it induces potentially digressive conclusions. We conclude that a binary segregation of relational assemblies into mechanisms and organisms is insufficient, and we indicate how a three-fold segregation throws new light on Rosen's model. An organism is not 'the complement of a mechanism': the complement of a mechanism is its ecosystem: an organism is the 'complex interface' between mechanism and ecosystem.

Keywords: Rosen; life; organisms; birationality; hyperscale

Introduction

Much of Robert Rosen's professional work targeted the development of relational biology and the way in which 'efficient cause' could be internalized in an organism. His book Life Itself (Rosen, 1991) focuses on precisely this aspect of living systems. Rosen constructed his relational model for an organism in terms of mathematical mappings. A mapping is a rule of correspondence between sets, which associates each element of a set with an element in the same or another set. In the notation usually used in Life Itself, small letters (e.g. 'm') are used to indicate elements of sets, which are denoted using capital letters (e.g. 'M'). A map is a function (e.g. f) such that for every element a of the domain A of the mapping there is a unique 'object' f(a) in the mapping's range B. It is important to note that for f to be correctly defined as a mapping, the relationships between elements a of A and elements b of B can be one-to-one and many-to-one, but not one-to-many. In technical terms, this means that a processor is permitted to
associate a single output with a number of different inputs (usually referred to in computing as a *fanin* greater than one) but *not* to generate a number of equivalent outputs from a single input (a *fanout* greater than one). We shall see later that this is an extremely restrictive omission from a model of an organism.

Rosen wished to be very clear that he was referring to two different relationships in his diagrams or graphs, and consequently settled on the device of using different arrows to indicate each of the relationships. Similarly, in what follows, a hollow-headed arrow denotes an actual software flow through hardware from input to output, and a solid-headed arrow denotes the induction or generation of that flow (see Figure 1).

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“The hollow-headed arrow denotes the actual software flow from input (afferent) to output (efferent)”

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“The solid-headed arrow denotes the induction or generation of this flow by the hardware”

**Figure 1:** Rosen's relational arrows as they are presented in *Life Itself*, and also in the form in which they will appear in this paper, where for added clarity we have added in distinguishing colors corresponding (apparently) to those in Rosen's original manuscript. The textual indications are taken from *Life Itself*, pp. 219-220.

Rosen's graphical representation of a mapping \( f \) from \( A \) to \( B \) consequently has the form illustrated in Figure 2. We must, however, be very careful in our interpretation of Rosen's graphs to distinguish between the hardware component or processor, denoted by \( f \), and the mapping it induces, also denoted by \( f \) (see, for example, *Life Itself*, pp. 221 and 249).

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**Figure 2:** Rosen's graphical representation of a mapping \( f \) from \( A \) to \( B \) by a single component or processor.

Unfortunately, editorial errors *Life Itself* lead to a degree of confusion, most particularly in the case of figure 10C.5, where all of Rosen's relational arrows were interchanged (hollow for solid; solid for hollow) during the editorial process (see Figure 3).
Rosen's relational model of an organism begins with a single component, or processor, \( f \) which produces outputs in \( B \) from inputs in \( A \), as illustrated in Figure 2. Rosen decides to refer to such a diagram as *metabolic*, and to \( f \) as the 'metabolic processor'. The processor \( f \) in Figure 2 is, however, problematic, as it has to be 'created' or *entailed* from outside the system illustrated in Figure 2. Rosen's intention was to construct a model within which every component of the system is 'created' or *entailed from within the model itself*, that is, so efficient cause of the system is completely internalized. By adding in a component \( \Phi \) to entail \( f \), which Rosen refers to as *repair*, and a component \( \beta \) to entail \( \Phi \), which he refers to as *replication*, he arrives at his figure 10C.5 - shown in its corrected form in Figure 3(b), above. The critical step was his realization that \( \beta \) did not need to be a new vertex of the graph, but that it could be derived from the elements of \( B \), by assuming that \( b \) is itself a processor. As Louie (2005) states, the condition under which this derivation is possible "... may be regarded as the abstract version of the one-gene-to-one-enzyme hypothesis."

The result of internalizing the function of \( \beta \) as illustrated in Figure 4, rather than imposing it from outside as in Figure 3(b), is that "In effect, the function of replication could be, under certain formal circumstances, already entailed by the two prior functions of metabolism and repair" (Life Itself, p. 251). The internalization of \( \beta \) yielded the well-known figure 10C.6 of *Life Itself*, illustrated in Figure 4 of this paper, below. Rosen maintained that any material system which realizes the graph or abstract block diagram that this figure represents is an organism.

Louie (2005) has correctly pointed out that there is a degree of inconsistency in the notation.
which is used in *Life Itself*, but if we replace its 'morphism: domain→codomain' mapping by his 'element-chasing' version (Louie, 2005, Figure 11) we apparently end up with an 'organism' which depends on no more than one gene: the majority of life's complexity, to some extent hinted at by Rosen's notation, disappears in the formalization. A major problem is to see where in Rosen's figure 10C.6 the multiplicity of genes appears. If we take 'as Gospel' that small letters (e.g. 'a') represent elements of sets (e.g. A), then there only appears to be one processor f which performs all the metabolic changes between A and B. On the other hand, in both Rosen's 'mixed notation' figures and Louie's 'element-chasing' figures, the *repair* functor is always symbolized by a capital Φ. Is this a single element, as it would appear to be from the 'element-chasing' name? Or is it a set, through whose influence the genome enters into the system? Rosen appears to state that capital F represents a single element, as does Wolkenhauer (2001, page 265).

A further question arises in respect of the use of formal mathematical mapping in the model. Recent research indicates that complete knowledge of the human genome is insufficient to determine human fabrication (Gravely, 2001; Mattick, 2003). Is the mathematical mapping of sets, with its restriction to one-to-one and many-to-one relationships, sufficient to describe biological processes, most particularly those relating to the complexity of genetic networks, to protein gene-switching, and to non-protein RNA-based catalysis?

**Natural Hierarchy and Birationality**

Our own studies of natural hierarchy are persuasive that both organisms and rationality itself constitute hyperscalar systems, and that they *always* operate within birational frameworks of 'entity and ecosystem' (see, for example, Cottam et al, 2003 and 2004 for a summary of the properties of natural hierarchy). How does this relate to Rosen's scheme of internalized 'efficient cause', which is apparently mono-scalar and mono-rational? Rosen intentionally eliminates 'environmental' influences from his model (see the caption to Figure 4, above). Is this feasible in a natural multi-scalar system, let alone a birational one? His relational description in terms of “entailment without states” rejects the implications of specific embodiment, whereas a birational hierarchy depends on the 'assimilation' of both interrelations and embodiment.

To the best of our knowledge, Rosen never delved deeply into questions of hierarchy with respect to living systems. Louie (2006) has stated that "There are three mappings in an (M,R)-system on three hierarchical levels, and they entail one another in a cyclic permutation." It is not clear how this would work if the word hierarchy, as Louie uses it, means other than solely a processor's control of a mapping, and therefore 'superiority' over its domain and range. It is worth noting that, if there is a rational pathway between different scalar levels of a hierarchy, then it will collapse into a single level unless it is externally constrained not to do so (Cottam et al, 2004). In Rosen's model, such a constraint would place the entailment of structure and of any consequent relationships *outside* the system. The cyclic permutation of three hierarchical levels - let us call them P, Q, R - would presumably entail some set of operations relating (P-to-Q), (Q-to-R), (R-to-P) or (P-to-R), (R-to-Q), (Q-to-P). In either case, there will be two valid inter-level transitions between adjacent scales of the hierarchy and one questionable one where the transition jumps over a scale without affecting it.

Rosen did, in fact, make one very important reference to the properties of a real hierarchy in this context: *important* because it discusses the mathematical relationships, or lack of them, between
scales. Because, so far as we are aware, this is the only place in the literature that such a consideration appears, we will quote the passage verbatim from the page 416 of the Appendix to *Anticipatory Systems*. Rosen is describing the mathematics of a system which presents a number of different informational layers, mathematically coupled together. He states that "... given any layer of informational structure, we can pass to succeeding layers by mere differentiation, and to antecedent layers by mere integration. Thus, in particular, knowledge of any layer in this infinite array of laters determines all of them...". While mathematically valid, this cannot meaningfully describe of a real scalar hierarchy, as we pointed out above. However, he then states that

"... our ability to do all this depends in an absolutely essential way on the exactness of the differential forms which arise at every level of our web of informational interaction, and which relate each level to its neighbors. For instance, if the forms in (7.4.2) are not exact, there are no functions \( f(x_1, ..., x_n) \) whose differentials are given by (7.4.2), and hence no rate equations of the form (7.4.1). In fact, in such a situation, the simple relations between the levels in our web (namely, that each level is the differential of the preceding level and the integral of the succeeding one) breaks down completely; and levels become independent of each other, and must be posited separately. Thus, two systems could have the same activation-inhibition patterns, but vastly different agonist-antagonist patterns, and hence manifest entirely different behaviors." (Rosen, 1985) ... the italics are Rosen's.

Rosen describes precisely what happens in a real natural hierarchy, including the appearance of level-independence which brings about the truncation of observation of observation of observation of..., promoting its stabilization as the self-observation of consciousness - much as he describes for Newtonian Mechanics in *Life Itself*. However, as he is not taking into account hierarchical self-correlation within a birational 'entity and ecosystem' framework, he misses that the solution to local inter-scalar transit in such a system is derived from the hierarchy's global properties (Cottam et al, 2004), and that information can be obtained from the substantive inter-scalar regions in real systems to facilitate the otherwise impossible task of interrelating adjacent levels, through a generic form of quantum error correction (Cottam et al, 2003).

### Replicating Rosen's (M,R)-Systems

We present three 'replications' of Rosen's (M,R)-Systems. The first one draws a parallel between the non-recursively replicating system Rosen illustrated in his figure 10C.5 and the relationship between a computer program and its compiler. Its purpose is to reproduce Rosen's sequence of efficient causes as a set of nested environments. The second is far simpler, and its purpose is to draw attention to the minimal separation between the points in a generating or controlling sequence which are connected together when forming a loop to curtail infinity. The third 'replication' is a redrawing of Rosen's figure 10C.6, and its relationship to the properties of a natural birational entity-ecosystem hierarchy.

### A Nested System

Rosen's scheme of a sequence of relational arrows leading to the (M,R)-System of his figure 10C.5 can be 'replicated' as a nested abstract association between the hand-written code for a computer program, its compiler, and a resulting program. Both yield the same graph-theoretic
description. In the following, we refer to the functors by the names of their transfer functions.

We require a program $H(Y,Z)$ which takes as its input data array $Y$, and provides as output array $Z$. We hand-code a source $X$ for the program, and feed it to a compiler $\Phi(X, H(Y,Z))$, which generates $H$ (Figure 5(a)). We can make the system more self-contained if we now configure the program $H$ to generate the original hand-coded source $X$ automatically (Figure 5(b)), by putting $X = Z$; why not?

![Figure 5: (a) The starting point: source-code $X$ to compiler $\Phi$ to program $H$ to operation $H(Y,Z)$, and (b) the program is now configured to generate the compiler input $X$ (i.e. we put $X = Z$).](image)

OK so far, but $\Phi$ is still externally entailed, i.e. we still have to write the compiler. However, the solution is to hand - we just need to entail $\Phi$. But first, let us redraw our diagram to illustrate the nesting involved (Figure 6(a)). The compiler $\Phi$ is the environment within which program $H$ is generated. By nesting the current assembly within another generating environment $\beta$ as replicator we can entail $\Phi$ (Figure 6(b))

$$\Pi(H(Z, H(Y,Z)), H(H(Y,Z), H(Z, H(Y,Z))))$$

![Figure 6: (a) A nested representation of the looped compiler-program assembly of Figure 5(b), and (b) the looped compiler-program assembly nested in the following generating environment $\beta$.](image)

The development of an infinite nesting sequence of generating environments is now evident. The following generator as replicator would have as transfer characteristic $II(H(Z, H(Y,Z)), H(H(Y,Z), H(Z, H(Y,Z)))$ ... and so on. As the graph-theoretic description is the same as for Rosen's...
figure 10C.5, the same loop-back connection can be established, subject to the same constraint.

**A Triangular Loop**

Further 'replication' of Rosen's scheme is as a circulation, in a triangular loop of three independent components, where on each circuit the flow is boosted by a kick from the environment (Figure 7). It is noteworthy that the relevant characterization of an organism is in terms of three components – metabolism, repair and replication: a *two*-component system of this kind would necessarily succumb to dissipation.

![Figure 7: A triangular loop 'replication' of Rosen's figure 10C.6: on each circuit the system can gain 'momentum' from the environmental input $A$.](image)

It is easy to see from Figure 7 why we earlier objected to Louie's (2006) reference to a cyclic permutation of entailment mappings on three hierarchical levels. Although it is indeed possible in a hierarchical system to create a multi-level loop of entailment, this can only be done in a *birational* hierarchy, as this is the only structure in which it is possible to continuously move in a hierarchically-meaningful direction (e.g. always 'upwards' from a *controlled* level to a *controlling* one) and still be able to return to the level of departure! (the Dutch graphical artist M. C. Escher has drawn an analogous picture of the walls of a castle, where as you walk round and round you are *always* going up steps).

**A Figure-of-Eight Replication**

Rosen presents his replicating (M,R)-System in figure 10C.6 of *Life Itself* in a very asymmetric manner (see Figure 4). When rearranged, while keeping the relational topology constant, the figure has a very symmetric appearance (Figure 8(a)). Minor modification to the arrows between $B$ and $f$ – again maintaining the topology – reveals the figure-of-eight relational sequence shown in Figure 8(b).
Figure 8: (a) The result of a topologically-constant manipulation of the shape of figure 10C.6, and (b) the figure-of-eight relational sequence revealed by a second minor topology-maintaining manipulation.

In a natural hierarchy we cannot successfully fractionate \{functor and function\} – it consequently makes no sense to talk about \(f, b/B\) or \(\varphi/\Phi\) in isolation, and we must look at Rosen's solid- and hollow-headed arrows as related pairs, as Rosen himself recognized – as 'functors/functions' or 'operators/operations'. The three functors/functions, in terms of individual elements, are then

\[ \text{metabolism: } f \rightarrow a \rightarrow b, \quad \text{repair: } f \rightarrow \varphi \rightarrow b \quad \text{replication: } b \rightarrow f \]

In our 'figure-of-eight' replication of Rosen's scheme, metabolism is now an independent functor/function (outlying \(f \rightarrow a \rightarrow b\)), repair is again an independent functor/function (outlying \(f \rightarrow \varphi \rightarrow b\)), and replication is the central assimilation of both their outcomes (central \(b \rightarrow f\)). A fascinating aspect of Figure 8(b) is that the two sides of the figure are complementary: solid-headed arrows on one side are replaced by hollow-headed ones on the other, and vice versa. Figure 9 shows Rosen's figure 10C.6 where the arrows have been numbered for convenience.

Figure 9: A version of Rosen's figure 10C.6, where the individual arrows have been numbered for convenience.

Recognition in general of a single object implies the existence of not two but three separate
domains: the object, its ecosystemic environment and their interface: the bifurcating categorization of nature proposed by Rosen into the complement of mechanism and organism is insufficient. An organism is not 'the complement of a mechanism': the complement of a mechanism is its ecosystem. So, how can we interpret Figure 9? It is important in this context to note that a mechanism can 'contain' an organism, but more to the point that an organism can 'contain' a mechanism (e.g. *Life Itself*, Figures 9E.3, 10C.6). Metabolic operator/operation $f \rightarrow a \rightarrow b$ (arrows 5 and 6) falls into Rosen's category of mechanisms, where solid-headed arrow $f \rightarrow a$ is the induction of software flow $a \rightarrow b$ (compare this with Figure 2). Repair operator/operation $f \rightarrow \phi \rightarrow b$, however (arrows 2 and 3), is very different: it is the opposite or complement of a mechanism: hollow-headed arrow $f \rightarrow \phi$ is the induction of hardware flow $\phi \rightarrow b$.

The central region (arrows 1 and 4), the replication operator/operation $b \rightarrow f$, is the intimate association of *induction of software flow*, *induction of hardware flow*, and both *software* and *hardware flows* themselves! This intimate four-fold association of birational causes and effects only exists in an organism. An organism is an intimate (complementary) coupling between a mechanism and its ecosystem. An organism is not 'the complement of a mechanism': the complement of a mechanism is its *ecosystem*: an organism is the 'complex interface' between mechanism and ecosystem.

The new version of the *metabolic* functor/function, arrows 5 and 6, corresponds to the operation of mRNA as an agent between DNA and enzyme. The new version of the *repair* functor/function, arrows 2 and 3, appears to be a very strange beast (see Figure 10). It comprises a software flow which induces material change. *But this is precisely the function of repair!* It should be remembered that there are a large number of interlinked repair mechanisms in a cell, arranged in a complex network, but primarily their function is to take previously stored information (either from the DNA, or from a wide range of other locations) and use it to recreate either the material functors or to repair material mutations in the DNA itself.

![Figure 10: The ecosystemic complement of a mechanism: arrows 2 and 3 of Figure 9.](image)

The most fascinating region is between $b$ and $f$, where software flow and material induction run in parallel. This region acts out a material instantiation of genetic information; it both makes use of genes, and recreates them. It is the interface between mechanism and ecosystem, and closely resembles the complex information-exchange region between a scalar Newtonian potential well of a natural hierarchy and its locally-scaled complementarily-rational complex inter-scalar region. Rosen's model of an organism turns out to be birational!

**Dealing with Infinity**

Rosen points out that "there is a certain parallel between the invocation of replication above, and
Newton's Second Law. Both serve to stop an incipient infinite regress, in each case by making what happens at an early step in the regress depend on preceding steps" (Life Itself, p.251). This is true enough, but it is not the whole story: the result is to replace one infinite sequence by another, different one. In the replication of figure 10C.6 as a nested system, if the nesting were not somehow truncated it would reach out to infinity. This is avoided, not by simplistically chopping off the sequence, but by making the sequence loop back on itself to an earlier state. It is not now the structure which is infinitely repeated, it is the process which is repetitive. If some kind of 'preference' can be imposed on the process as it loops, then the possibility for evolution now exists. This illuminates the unfortunate nature of using formally mathematical mapping to model living systems: its exclusion of one-to-many relationships removes possibly multiple paths through the system, and greatly reduces the probability of mutation through inter-path conflict.

The most interesting aspect of this is what happens when some kind of change takes place. For the case of infinite nesting, ripples from a change spread out (or in) from the nested level which was perturbed. An observer at any specific nested level will feel an unexpected stimulus appear, then disappear as if it had never existed. The result for a looped nesting is quite different. The observer will still feel an unexpected stimulus appear, and disappear, but will then experience a series of 'echoes' as the perturbation goes round and round the loop, being changed, or attenuated, or even amplified as it progresses. If the observer is worth his (or her) salt, he (or, of course, she) can progressively build up a partial picture of the loop's characteristics, as modifications in the perturbation from occurrence, to reoccurrence, to reoccurrence, ... are a direct result of the multiple levels' propagation characteristics. The change from infinite to nested provides the observer with motivation to escape a nominally internalist position by modeling an outside world from regularities in his (or her) phenomenal observations. In some strange way it seems that this process of 'dealing with infinity by looping back to an earlier state' creates time through its generation of a recursive chronicle.

**Conclusion**

Rosen's model 'does what he wanted it to do'; it (almost) 'internalizes efficient cause'. But it is also hyperscalar, because everything is! His notational mixing of sets & elements is in fact superficially useful, in that it maintains at least an impression of implicit complexity. An organism is an intimate (complementary) coupling between a mechanism and its ecosystem. An organism is not 'the complement of a mechanism': the complement of a mechanism is its ecosystem; an organism is the 'complex interface' between mechanism and ecosystem. The next most useful step in validating Rosen's work will be to reformulate the multi-scalar generality of a hyperscalar natural hierarchy in terms of notional mappings, to see what happens when the formally-mathematical relationships are extended to include not only one-to-one and many-to-one relationships but also the one-to-many relationships which Rosen excluded. In the absence of mutation, one-to-one and many-to-one mappings preclude evolution: one-to-many does not. Will this crude 'injection' of Rosennan-style complexity into a notionally-mapped self-correlating natural hierarchy convert a merely complicated mechanistic network into an organism?

**References**


